Section 3.7

The Derivative Rule for Inverses

If f has an interval I as domain and $f'(x)$ exists and is never zero on I, then f^{-1} is differentiable at every point in its domain. The value of $\left(f^{-1}(x)\right)'$ at a point b in the domain of f^{-1} is the reciprocal of the value of $f'(x)$ at the point $a = f^{-1}(b)$:

$$\left(f^{-1}\right)'(b) = \frac{1}{f'\left(f^{-1}(b)\right)}$$

Or

$$\frac{df^{-1}}{dx} = \frac{1}{\frac{df}{dx}\Big|_{x=f^{-1}(b)}}$$

Example: Let $f(x) = \frac{1}{4}x^3 + x - 1$.

a) What is the value of $f^{-1}(3)$?
b) What is the value of $\left(f^{-1}\right)'(x)$ when x = 3?

Solution:
Recall: The domain of *f* is the range of f^{-1} and the range of *f* is the domain of f^{-1}. Therefore, we need to find the value of *x* for which *f* equals 3.

$$\frac{1}{4}x^3 + x - 1 = 3 \text{ when x = 2.}$$

So, $f^{-1}(3) = 2$.

Because the function is differentiable and has an inverse, we can use the above theorem.

$$\left(f^{-1}\right)'(3) = \frac{1}{f'\left(f^{-1}(3)\right)} = \frac{1}{f'(2)}$$

$$f' = \frac{3}{4}x^2 + 1$$
$$f'(2) = 4$$
$$\left(f^{-1}\right)'(3) = \frac{1}{4}$$