

Final Exam  
No Calculator!  
No Notes!

1. Find each derivative. You do not need to simplify. **5 points each**

a)  $y = \arctan(4 - x^2)$

b)  $y = \ln(\sec(x^2))$

c)  $y = \cos(\sin(e^{x^2}))$

d)  $y = \frac{x-2}{\sqrt{x^2+1}}$

e)  $\tan(xy) = \cos(x+y)$

f)  $y = (\sin x)^x$

2. Evaluate each integral. **5 points each**

a)  $\int \tan(3x - 1) dx$

b)  $\int \cos^2(2x)dx$

c)  $\int \arcsin x dx$

d)  $\int e^x \sin x dx$

e)  $\int x^3 e^{3x} dx$

5. Derive the derivative of  $y = \arctan(3x)$ . **5 points**

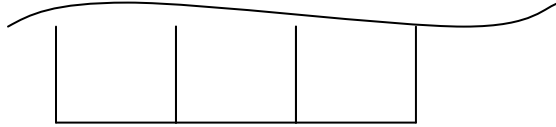
Math 180  
Winter, 2008

Name \_\_\_\_\_

Final Exam  
No work = no credit!

1. Find the area of the “triangular” region bounded on the left by  $x + y = 2$ , on the right by  $y = x^2$ , and above by  $y = 2$ . **8 points**

2. Three adjacent rectangular pens will be bounded on one side by a river. With 1000 meters of fencing, what is the largest area you can enclose, and what are the dimensions? **8 points**



3. Water runs into a conical tank at the rate of 9 cubic feet per minute. The tank stands point down and has a height of 10 feet and a base radius of 5 feet. How fast is the water level rising when the water is 6 feet deep? **8 points**

$$V = \frac{1}{3} \pi r^2 h$$

4. Find each of the following limits. **4 points each**

a)  $\lim_{x \rightarrow \pi} \frac{\tan x}{x}$

b)  $\lim_{x \rightarrow 0} \left( \frac{1}{\sin x} - \frac{1}{x} \right)$

c)  $\lim_{x \rightarrow 0^+} (1+x)^{\frac{1}{x}}$

d)  $\lim_{h \rightarrow 0} \frac{\sqrt{3-x-h} - \sqrt{3-x}}{h}$

5. The sum of two nonnegative numbers is 20. Find the numbers if the product of one number and the square root of the other is to be as large as possible. **8 points**

6. Let  $f(x) = x\sqrt{3-x}$ . **20 points**

Domain \_\_\_\_\_ Range \_\_\_\_\_

Intercepts \_\_\_\_\_ Critical point(s) \_\_\_\_\_

Local Maximum(s) \_\_\_\_\_ Local Minimum(s) \_\_\_\_\_

Inflection point(s) \_\_\_\_\_

Intervals where the graph of  $f$  is increasing \_\_\_\_\_

Intervals where the graph of  $f$  is decreasing \_\_\_\_\_

Intervals where the graph of  $f$  is concave up \_\_\_\_\_

Intervals where the graph of  $f$  is concave down \_\_\_\_\_

$f'$

$f''$

Sketch the graph.

7. Find the area of the region enclosed by  $x^3 = y$  and  $3x^2 - y = 4$ .

8. Find an equation of the tangent line and the normal line to the graph of the curve  $y = \frac{x}{x^2 - 4}$  at the point  $\left(1, -\frac{1}{3}\right)$ . **8 points**