

Math 180
Winter, 2008

Name _____

Exam 1 Review
No Work = No Credit!

1. Evaluate each of the limits.

a) $\lim_{t \rightarrow 9} \frac{\sqrt{t} - 3}{t - 9}$

b) $\lim_{x \rightarrow 0^-} \frac{|x|}{x}$

c) $\lim_{x \rightarrow 0} \frac{\sin(7x)}{\sin(3x)}$

$$\text{d) } \lim_{x \rightarrow \infty} \frac{4x^2 - x}{2x^3 - 5}$$

$$\text{e) } \lim_{x \rightarrow \infty} \frac{\sqrt{x^2 + 2}}{3x - 6}$$

$$\text{f) } \lim_{x \rightarrow \infty} \frac{e^x + e^{-x}}{e^x - e^{-x}}$$

2. Use the δ/ε definition of the limit to prove that $\lim_{x \rightarrow 2} (4x - 7) = 1$.

3. Determine if the following function is an even function, an odd function, or neither.

$$f(x) = \frac{x^5 - x}{1 + x^2}$$

4. Find a nonzero value for the constant f that makes

$$f(x) = \begin{cases} \frac{\tan(kx)}{x}, & x < 0 \\ 3x + 2k^2, & x \geq 0 \end{cases}$$

continuous at $x = 0$.

5. Let $f(x) = \frac{1}{1-x}$.

- a) What are the domain and the range of f ?
- b) Find $f \circ f$ and its domain.
- c) Find $f \circ f \circ f$ and its domain.

6. A right triangle is formed in the first quadrant by the x- and y-axes and a line through the point (3, 2). Write the length of the hypotenuse as a function of x.

7. Verify that the Intermediate Value Theorem applies to the indicated interval and then find the zero guaranteed by the Theorem.

$$f(x) = x^3 - x^2 + x - 2$$

$$[0, 3]$$

8. Let $f(x) = \frac{1}{x+1}$.

- a) Find all horizontal asymptotes using limits.
- b) Find all vertical asymptotes using limits.
- c) Find the domain.
- d) Sketch the graph of f .
- e) Find the range.
- f) Find an equation of the tangent line to the graph of f at the point $(0,1)$.
- g) Sketch the graph of the tangent line on the same set of axes as part (d).

9. Ten feet of wire is to be used to form a square and a circle. Write a function that represents the total area of both figures as a function of x , the point where the wire is cut. State the domain of the function.



10. True False If $\lim_{x \rightarrow c} f(x) = L$, then $f(c) = L$.
11. True False If f is undefined at $x = c$, then $\lim_{x \rightarrow c} f(x) \text{ dne}$.
12. True False If f is continuous at $x = c$, then f is differentiable at $x = c$.
13. True False If $f(x) = g(x)$ for all real numbers except $x = 0$, and $\lim_{x \rightarrow 0} f(x) = L$, then $\lim_{x \rightarrow 0} g(x) = L$.
14. True False If $\lim_{x \rightarrow c} f(x) = L$ and $f(c) = L$, then f is continuous at $x = c$.

15. Let $f(x) = \frac{|x^2 - 4|}{x - 2}$.

Find each of the following.

a) Domain of f .

b) $\lim_{x \rightarrow 2^-} f(x)$

c) $\lim_{x \rightarrow 2^+} f(x)$

d) $\lim_{x \rightarrow 2} f(x)$

e) Is f continuous at $x = 2$? Why or why not?

f) Sketch the graph of f .

16. Find $\lim_{x \rightarrow 0} x^2 \sin\left(\frac{1}{x}\right)$. Use the Sandwich Theorem.

17. Use the limit definition of derivative to find the derivative of $f(x) = -x^2 + 3x - 2$.