

How to Rationalize Denominators

1. If the denominator is a single term and a square root:

$$\text{Multiply by } 1 = \frac{\text{denominator}}{\text{denominator}}$$

Example:

$$\frac{3}{\sqrt{5}} = \frac{3}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{3\sqrt{5}}{5}$$

2. If the denominator is a single term and not a square root:

Multiply by 1 but the way we write 1 depends on the problem.

Example

$$\frac{1}{\sqrt[3]{x}} = \frac{1}{x^{\frac{1}{3}}} = \frac{1}{x^{\frac{1}{3}}} \cdot \frac{x^{\frac{2}{3}}}{x^{\frac{2}{3}}} = \frac{x^{\frac{2}{3}}}{x^{\frac{1}{3} + \frac{2}{3}}} = \frac{\sqrt[3]{x^2}}{x}$$

Example:

$$\frac{1}{\sqrt[4]{8}} = \frac{1}{\sqrt[4]{2^3}} = \frac{1}{\sqrt[4]{2^3}} \cdot \frac{\sqrt[4]{2}}{\sqrt[4]{2}} = \frac{\sqrt[4]{2}}{\sqrt[4]{2^4}} = \frac{\sqrt[4]{2}}{2}$$

Goal: Make the exponent and the root the same power so that you have $\sqrt[n]{b^n}$ in the denominator.

3. If the denominator is a sum or a difference:

$$\text{Multiply by } 1 = \frac{\text{conjugate}}{\text{conjugate}}$$

All a conjugate does is change a sum to a difference or
Change a difference to a sum.

Example:

$$\frac{1}{3+\sqrt{5}} = \frac{1}{3+\sqrt{5}} \cdot \frac{3-\sqrt{5}}{3-\sqrt{5}} = \frac{3-\sqrt{5}}{9-3\sqrt{5}+3\sqrt{5}-5} = \frac{3-\sqrt{5}}{4}$$

The conjugate of $a+b$ is $a-b$.

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$$(a+b)(a-b) = a^2 - b^2.$$