Math 71

Chapter Notes

A **relation** is any set of ordered pairs. The set of the first components of the ordered pairs is called the **domain** of the relation and the set of all second components is called the **range** of the relation.

A **function** is a relation in which no two ordered pairs have the same first component and different second components. A **function** is a correspondence (a rule) from a first set, called the **domain**, to a second set, called the **range**, such that each element in the domain corresponds to exactly one element in the range. For each value of *x*, there is one and only one *y*. The variable *x* is called **independent** variable because it can be assigned any value from the domain. The variable *y* is called the **dependent** variable because its value depends on *x*.

The notation $f(x)$**,** read *"f of x"*, represents the value of the function at the number *x*.

Vertical Line Test
If any vertical line intersects a graph in more than one point, the graph does not define *y* as a function of *x*.

Domain
When asked to find the domain of a function, ask yourself these questions.

1. Does the expression contain a rational expression (a fraction)? If so, the denominator cannot be zero.
   Set denominator = 0 and solve the resulting equation.
   The values of *x* that make the denominator = 0 cannot be in the domain of the function.

2. Does the expression contain a radical expression with an even root? If so, the radicand (the expression under the radical) must be greater than or equal to zero.
   Solve the inequality:
   $radicand \geq 0$
   The values of *x* that solve the inequality form the domain of the function.

3. Does the expression contain a logarithm? If so, the argument of the logarithm must be greater than zero.
   Solve the inequality:
   $\arg ument > 0$
   The values of *x* that solve the inequality form the domain of the function.

4. If the answer to the above three questions is "**no"**, and the problem is not an application problem, the domain is that *x* can be any reall number.

Let $f(x)=\dfrac{3x-1}{x-5}$.

a) State the domain.

Find the indicated function values.

b) $f(0)$

c) $f(3)$

d) $f(-3)$

e) $f(a+h)$

Let *f* and *g* be two functions.

1. $(f+g)(x)=f(x)+g(x)$
2. $(f-g)(x)=f(x)-g(x)$
3. $(fg)(x)=f(x)\cdot g(x)$
4. $\left(\dfrac{f}{g}\right)(x)=\dfrac{f(x)}{g(x)}, g(x)\neq 0$

Let $f(x)=x^2+4x$ and $g(x)=2-x$. Find each of the following.

a) $(f+g)(x)$

b) $(f+g)(4)$

c) $(f-g)(x)$

d) $(f-g)(-2)$

e) $(fg)(x)$

f) $(fg)(-3)$

g) $\left(\frac{f}{g}\right)(x)$

h)the domain of $\left(\frac{f}{g}\right)(x)$

All equations of the form $Ax+By=C$ are **linear equations**. The graph of a linear equation is a straight line. To find the *x-intercept*, set *y = 0*. To find the *y-intercept*, set *x = 0*.

The **slope** of a line is a measure of the steepness of the line. The slope of the line through two distinct points (x_1, y_1) and (x_2, y_2) is

$$m=\frac{rise}{run}=\frac{\Delta y}{\Delta x}=\frac{change\ in\ y}{change\ in\ x}$$

The **slope-intercept equation** of a non-vertical line with slope *m* and y-intercept *b* is

$$y=mx+b$$

The **point-slope equation** of a non-vertical line with slope **m** containing point (x_1, y_1) is

$$y - y_1 = m(x - x_1)$$

$5x + 3y = 15$

Find the x-intercept.

Find the y-intercept.

Graph the line.

Solve the equation for *y*.

State the slope of the line.

Graph the line using the slope intercept form.

As a craftsman making feather dusters, you have invested $250 in materials. You expect to sell your dusters for $7.50 each.

a) Set up a model, draw a graph, and determine an equation to calculate your profit.
b) Use your graph to estimate how many dusters you would have to sell to break even.
c) How many dusters would you have to sell to earn a profit of at least $100?

If two lines are parallel, the two lines have the same slope.
If two lines are perpendicular, the two lines have slopes that are negative reciprocals of each other.

$$l_1 \parallel l_2 \leftrightarrow m_1 = m_2$$
$$l_1 \perp l_2 \leftrightarrow m_1 = -\frac{1}{m_2}$$

Find an equation of the line that passes through the points $(2,1)$ and $(-1,-5)$. Graph the line.

Find an equation of the line whose slope is undefined and contains the point $(3,-1)$.
Graph the line.

How to Find an Equation of a Line

1. Find the slope *m*.
    a. If given two points, use the slope formula to find the slope.
    b. The slope could be given.
    c. If the line you need is parallel to another line, then the slopes of the two lines are equal.
    d. If the line you need is perpendicular to another line, then the slope of the line you need is the negative reciprocal of the slope of the given line.
2. Select a point (x_1, y_1).
3. Substitute the slope *m* and the point (x_1, y_1) into the point slope form of the line.
    $y - y_1 = m(x - x_1)$
4. Do the algebra to get the line into the point intercept form of the line.
    $y = mx + b$

Find an equation of the line that is parallel to the line $y = 2x - 1$ and contains the point $(-2, 3)$. Graph both lines.

Find an equation of the line that is perpendicular to the line $x - 2y = 4$ and contains the point $(1, 3)$. Graph both lines.