



## Solving an Applied Problem

**Step 1:** Read the problem carefully until you understand what is given and what is to be found.

**Step 2:** Assign a variable to represent the unknown value, using diagrams or tables as needed. Write down what the variable represents. If necessary, express any other unknown values in terms of the variable.

**Step 3:** Write an equation using the variable expression(s).

**Step 4:** Solve the equation.

**Step 5:** State the answer in a complete sentence. Does the answer make sense? Is it reasonable?

**Step 6:** Check the answer in the words of the original problem.

### Example 1

If four times a number is added to 8, the result is three times the number added to 5. Find the number.

**Step 1:** Read the problem carefully.

**Step 2:** Assign a variable to represent the unknown variable. Look at the last statement in the problem – find the number.

Let  $x$  = the number

**Step 3:** Write an equation.

$$4x + 8 = 3x + 5$$

**Step 4:** Solve the equation.

$$4x + 8 = 3x + 5$$

$$x = -3$$

**Step 5:** State the answer.

The number is  $-3$ .

**Step 6:** Check your answer.

$$4(-3) + 8 = 3(-3) + 5$$

$$-12 + 8 = -9 + 5$$

$$-4 = -4$$

### Example 2

The total number of Democrats and Republicans in the U.S. House of Representatives during the 106<sup>th</sup> session was 434. There were 12 more Republicans than Democrats. How many members of each party were there?

**Step 1:** Read the problem carefully.

**Step 2:** Assign a variable to represent the unknown variable.

There were 12 more Republicans than Democrats. The number of Republicans is given in terms of the number of Democrats. Therefore,

Let  $x$  = # of Democrats

Then,

$x + 12$  = # of Republicans

Step 3: Write an equation.

**The total number of Democrats and Republicans in the U.S. House of Representatives during the 106<sup>th</sup> session was 434.**

$$x + x + 12 = 434$$

$$2x + 12 = 434$$

Step 4: Solve the equation.

$$x + x + 12 = 434$$

$$2x + 12 = 434$$

$$2x = 422$$

$$x = 211$$

$$x + 12 = 223$$

Step 5: State the answer.

The number of Democrats is 211, while the number of Republicans is 223.

Step 6: Check your answer.

$$211 + 223 = 434$$

### Example 3

The largest sheep ranch in the world is located in Australia. The number of sheep on the ranch is  $\frac{8}{3}$  the number of uninvited kangaroos grazing on the pastureland. Together, herds of these two animals number 88,000. How many sheep and how many kangaroos roam the ranch?

Step 1: Read the problem.

Step 2: Assign a variable to represent the unknown value.

The number of sheep on the ranch is  $\frac{8}{3}$  the number of uninvited kangaroos grazing on the pastureland. The number of sheep is given in terms of the number of kangaroos. Therefore,

Let  $x = \#$  of kangaroos

Then,

$$\frac{8}{3} \cdot x = \# \text{ of sheep}$$

Step 3: Write an equation.

**herds of these two animals number 88,000**

$$x + \frac{8}{3} \cdot x = 88000$$

Step 4: Solve the equation.

$$x + \frac{8}{3} \cdot x = 88000$$

$$\frac{11}{3} \cdot x = 88000$$

$$3 \cdot \frac{11}{3} \cdot x = 88000 \cdot 3$$

$$11x = 264000$$

$$x = 24000$$

$$\frac{8}{3} \cdot x = 64000$$

Step 5: State the answer.

Kangaroos number 24,000 and sheep number 64,000.

Step 6: Check the answer. Does it make sense?

The only reasonable answer to this problem would be whole numbers—no fractions or negative numbers.

## Example 4

In his job at the post office, Eddie works a 6.5-hr day. He sorts mail, sells stamps, and does supervisory work. One day he sold stamps twice as long as he sorted mail, and he supervised .5 hr longer than he sorted mail. How many hours did he spend at each task?

Need to find the number of hours he sold stamps, the number of hours he sorted mail, and the number of hours he supervised.

One day he sold stamps twice as long as he sorted mail, and he supervised .5 hr longer than he sorted mail. Note that the number

of hours he sold stamps and the number of hours he supervised are given in terms of the number of hours he sorted mail. Therefore,

Let  $x$  = # of hours he sorted mail

Then,

$2x$  = # of hours he sold stamps

$x + .5$  = # of hours he supervised

$$x + 2x + x + .5 = 6.5$$

$$4x + .5 = 6.5$$

$$4x = 6$$

$$x = 1.5$$

$$2x = 3$$

$$x + .5 = 2$$

Eddie sorted mail for 1.5 hours, sold stamps for 3 hours, and supervised for 2 hours.

## Example 5

Find the measure of an angle such that the sum of the measures of its complement and its supplement is  $160^\circ$ .

Let  $x$  = measure of the angle

$90 - x$  = measure of its complement

$180 - x$  = measure of its supplement

sum of the measures of its complement and its supplement is  $160^\circ$ .

$$90 - x + 180 - x = 160$$

$$270 - 2x = 160$$

$$-2x = -110$$

$$x = 55$$

The angle measures  $55^\circ$ .

## Example 6

If the first and third of three consecutive even integers are added, the result is 22 less than three times the second integer. Find the integers.

Let  $x$  = first even integer

$x + 2$  = second consecutive even integer

$x + 4$  = third consecutive even integer

first and third of three consecutive even integers are added, the result is 22 less than three times the second integer

$$x + x + 4 = 3(x + 2) - 22$$

$$2x + 4 = 3x + 6 - 22$$

$$2x + 4 = 3x - 16$$

$$-x = -20$$

$$x = 20$$

$$x + 2 = 22$$

$$x + 4 = 24$$

The even integers are 20, 22, and 24.

## Example 7

If sales tax on a \$16.00 compact disc is \$1.32, how much would the sales tax be on a \$120.00 compact disc player?

Let  $x$  = sales tax on the \$120 CD player

$$\frac{1.32}{16} = \frac{x}{120}$$

$$1.32(120) = 16x$$

$$158.40 = 16x$$

$$9.90 = x$$

The amount of sales tax on a \$120 CD player is \$9.90.

In this problem, we used proportions to solve this problem. A proportion says that two ratios are equal.

## Example 8

Solve:  $\frac{m}{5} = \frac{m-2}{2}$

$$\frac{m}{5} = \frac{m-2}{2}$$

$$2m = 5(m-2)$$

$$2m = 5m - 10$$

$$-3m = -10$$

$$m = \frac{10}{3}$$

## Example 9

How many liters of 25% acid solution must be added to 80 liters of 40% solution to get a solution that is 30% solution?

Strength	Amount	$S \times A$
•		*

- The number placed here cannot be the largest or the smallest number.

\* The number placed here can be attained two ways.

=====

These two lines mean to sum.

Strength	Amount	$S \times A$
.25		
.40		
.30		

80 liters of 40%

Strength	Amount	$S \times A$
.25		
.40	80	
.30		

How many liters of 25% acid?

Strength	Amount	$S \times A$
.25	$x$	$.25x$
.40	80	$.4(80) = 32$
.30	$x + 80$	$.30(x + 80)$

$$.25x + 32 = .30(x + 80)$$

$$.25x + 32 = .30x + 24$$

$$8 = .05x$$

$$160 = x$$

160 liters of a 25% solution would be needed.

Does the answer make sense?

Look at the table. 25% is closer to 30% than 40% is – therefore, we would need more liters of the 25% solution. In fact, 25% is 5% away from 30% while 40% is 10% away, twice as far away. Look, we need 160 liters of the 25% solution, twice as many liters as the 40% solution.

## Example 10

A coin collector has \$1.70 in dimes and nickels. She has 2 more dimes than nickels. How many nickels does she have?

Let  $x$  = # of nickels  
 $x + 2$  = # of dimes

The amount of money the coin collector has in nickels is  $.05x$ ; the amount of money the coin collector has in dimes is  $.10(x + 2)$ . Therefore, the total amount of money is

$$.05x + .10(x + 2) = 1.70$$

$$.05x + .10x + .20 = 1.70$$

$$.15x = 1.50$$

$$x = 10$$

$$x + 2 = 12$$

The coin collector has 10 nickels.

## Example 11

LaShondra inherited some money from her uncle. She deposited part of the money in a savings account paying 2%, and \$3000 more than that amount in a different account paying 3%. Her annual interest income was \$690. How much did she deposit at each rate?

Let  $x$  = \$ invested at 2%

$x + 3000$  = \$ invested at 3%

$$I = prt$$

$I$  = interest

$p$  = amount invested

$r$  = interest rate

$t$  = time

Interest earned at 2% is  $.02x$ .

Interest earned at 3% is  $.03(x + 3000)$

Total interest earned is \$690.

$$.02x + .03(x + 3000) = 690$$

$$.02x + .03x + 90 = 690$$

$$.05x = 600$$

$$x = 12000$$

$$x + 3000 = 15000$$

LaShondra invested \$12,000 @ 2% and \$15,000 @ 3%.

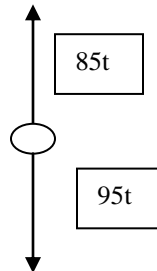
## Example 12

A train leaves Kansas City, Kansas, and travels north at 85 km per hour. Another train leaves at the same time and travels south at 95 km per hour. How long will it take before they are 315 km apart?

Let  $t$  = # hours both trains travel

$85t$  = distance traveled by first train

$95t$  = distance traveled by second train



The total distance traveled is 315 km..

$$85t + 95t = 315$$

$$180t = 315$$

$$t = 1\frac{3}{4}$$

The trains will be 315 km apart in  $1\frac{3}{4}$  hours.

## Example 13

Two cars start from towns 400 miles apart and travel toward each other. They meet after 4 hours. Find the speed of each car if one travels 20 mph faster than the other.

Let  $x$  = speed of one of the cars

$x + 20$  = speed of the other car

$$d = rt$$

$$\text{distance} = \text{rate} \cdot \text{time}$$

The first car traveled  $4x$  miles. The other car traveled  $4(x + 20) = (4x + 80)$  miles.

The total miles driven by both cars is 400 miles.

$$4x + 4x + 80 = 400$$

$$8x = 320$$

$$x = 40$$

One car traveled at 40 mph, while the other car traveled at 60 mph.