Math 71

Chapter 11 Notes

A **sequence** is a function whose domain is the first *n* positive integers. The function values or **terms** of the sequence are represented by

$$a_1, a_2, \cdots, a_n$$

$a_1 =$ 1st term in the sequence
$a_2 =$ 2nd term in the sequence
a_n = nth term or general term in the sequence

Example: Write the 1st 4 terms of the sequence whose general term is given.

$$a_n = \frac{2n}{n+4}$$

Products of consecutive positive integers occur quite often in sequences. These products can be expressed in a special notation, called **factorial notation.**

If *n* is a positive integer, the notation *n!* (read *n* factorial) is the product of all positive integers from *n* down through 1.

$$5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$$
$$6! = 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$$
$$0! = 1$$

The sum of the first *n* terms of a sequence is represented by the **summation notation.**

$$\sum_{i=1}^{n} a_i = a_1 + a_2 a_3 + \cdots + a_n$$

where *i* is the index of summation, *n* is the upper limit of summation, and 1 is the lower limit of summation.

Example:

$$\sum_{i=1}^{8} i = 1+2+3+4+5+6+7+8 = 36$$

Example: Find the indicated sum.

$$\sum_{i=1}^{5} \frac{i!}{(i-1)!}$$

Express the sum using summation notation.

$$2+4+6+\cdots+30$$

$$1+3+5+7+\cdots+37$$

An **arithmetic sequence** is a sequence in which each term after the first differs from the preceding term by a constant amount. The difference between consecutive terms is called the **common difference** of the sequence.

Example:

$$3, 8, 13, 18, ...$$

$$-10, -4, 2, 8...$$

The *nth* term (the general term) of an arithmetic sequence with first term a_1 and common difference *d* is

$$a_n = a_1 + (n-1)d$$

Example: Find the eighth term of the arithmetic sequence whose first term is -4 and whose common difference is 8.

The sum of the first *n* terms of an arithmetic sequence, denoted by S_n, and called the ***n* partial sum**, is given by

$$S_n = \frac{n}{2}(a_1 + a_n)$$

Example: Find the sum of the first 25 terms of the arithmetic sequence

$$7, 19, 31, 43, ...$$

Example: Find the sum of

$$2+4+6+\cdots+50$$

Example: A theater has 30 seats in the first row, 32 seats in the second row, increasing by 2 seats each row for a total of 26 rows. How many seats are there in the theater?

A **geometric sequence** is a sequence in which each term after the first is obtained by multiplying the preceding term by a fixed nonzero constant. The amount by which we multiply each time is called the **common ratio** of the sequence.

Example:

$$5, 10, 20, 40, \ldots$$

$$4, \frac{8}{3}, \frac{16}{9}, \frac{32}{27}, \ldots$$

The ***nth term*** **(general term)** of a geometric sequence with the first term a_1 and common ratio *r* is

$$a_n = a_1 r^{n-1}$$

Example: Find the 12th term of the geometric sequence with 4 as the first term and a ratio of -2.

The sum of the first *n* terms of a geometric sequence, denoted by S_n, and called the ***n*th partial sum,** is given by

$$S_n = \frac{a_1(1-r)^n}{1-r}, \quad r \neq 1$$

An infinite sum of the form $a_1 + a_1 r + \cdots + a_1 r^{n-1} + \cdots$ is called an **infinite geometric series.**

If $-1 < r < 1$, then the sum of the infinite geometric series is given by

$$S_\infty = \frac{a_1}{1-r}.$$

Example: Find the sum of the first 14 terms of the geometric sequence

$$-\frac{1}{24}, \frac{1}{12}, \frac{-1}{6}, \frac{1}{3}, \ldots$$

Example: Find the sum of the infinite geometric series.

$$3 - 1 + \frac{1}{3} - \frac{1}{9} + \ldots$$

Example: Express the repeating decimal as a fraction in lowest terms.

0.47474747...

Example: A skydiver falls 16 feet during the first second of a dive, 48 feet during the second second, 80 feet during the third second, 112 feet during the fourth second, and so on. Find the distance that the skydiver falls during the 15^{th} second and the total distance the skydiver falls in 15 seconds.