Math 181

Trig Substitutions

Expression	Substitution	Domain
$\sqrt{a^2 - x^2}$	$x = a\sin\theta$	$-\pi/2 \le \theta \le \pi/2$
$\sqrt{a^2 + x^2}$	$x = a\tan\theta$	$-\pi/2 < \theta < \pi/2$
$\sqrt{x^2 - a^2}$	$x = a\sec\theta$	$0 \le \theta < \pi/2 \ or\ \pi \le \theta < 3\pi/2$

Special Integration Formulas (a > 0)

$$\int \sqrt{a^2 - u^2}\,du = \frac{1}{2}\left(a^2 \arcsin\frac{u}{a} + u\sqrt{a^2 - u^2}\right) + C$$

$$\int \sqrt{u^2 - a^2}\,du = \frac{1}{2}\left(u\sqrt{u^2 - a^2} - a^2 \ln\left|u + \sqrt{u^2 + a^2}\right|\right) + C, u > a$$

$$\int \sqrt{u^2 + a^2}\,du = \frac{1}{2}\left(u\sqrt{u^2 + a^2} + a^2 \ln\left|u + \sqrt{u^2 + a^2}\right|\right) + C$$

Math 181

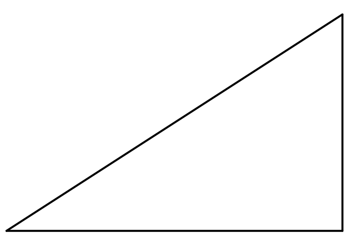
Integrate $\int \frac{dx}{\sqrt{4x^2+1}}$.

Solution:

Let $u = 2x$, $a = 1$, *and* $2x = \tan\vartheta$. Then

$$dx = \frac{1}{2}\sec^2\vartheta d\vartheta$$
$$2x = \tan\vartheta$$
$$4x^2 = \tan^2\vartheta$$
$$4x^2 + 1 = \tan^2\vartheta + 1 = \sec^2\vartheta$$
$$\sqrt{4x^2+1} = \sec\vartheta, -\pi/2 < \vartheta < \pi/2$$

$$\begin{aligned}\int \frac{1}{\sqrt{4x^2+1}}dx &= \frac{1}{2}\int \frac{\sec^2\vartheta}{\sec\vartheta}d\vartheta \\ &= \frac{1}{2}\int \sec\vartheta d\vartheta \\ &= \frac{1}{2}\ln\left|\sec\vartheta + \tan\vartheta\right| + C\end{aligned}$$



$$\sec\vartheta = \sqrt{4x^2+1}$$
$$\tan\vartheta = 2x$$

$$\int \frac{1}{\sqrt{4x^2+1}}dx = \frac{1}{2}\ln\left|\sqrt{4x^2+1} + 2x\right| + C$$

Math 181

Integrate $\int \frac{1}{(x^2+1)^{3/2}}\,dx$.

Solution:

$$(x^2+1)^{3/2} = \left(\sqrt{x^2+1}\right)^3$$

$$Let\ u = x = \tan\vartheta$$

$$a = 1$$

$$dx = \sec^2\vartheta\, d\vartheta$$

$$\sqrt{x^2+1} = \sec\vartheta$$

Then

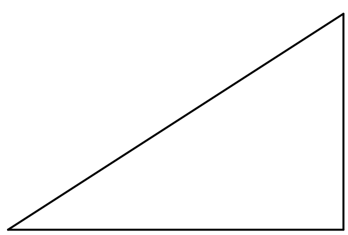
$$\int \frac{1}{(x^2+1)^{3/2}}\,dx = \int \frac{1}{\left(\sqrt{x^2+1}\right)^3}\,dx$$

$$= \int \frac{\sec^2\vartheta}{\sec^3\vartheta}\,d\vartheta$$

$$= \int \frac{d\vartheta}{\sec\vartheta}$$

$$= \int \cos\vartheta\, d\vartheta$$

$$= \sin\vartheta + C$$



$$\sin\vartheta = \frac{x}{\sqrt{x^2+1}}$$

$$\int \frac{1}{(x^2+1)^{3/2}}\,dx = \frac{x}{\sqrt{x^2+1}} + C$$

Math 181

Special Integration Formulas (a > 0)

$$\int \sqrt{a^2 - u^2}\, du = \frac{1}{2}\left(a^2 \arcsin\frac{u}{a} + u\sqrt{a^2 - u^2} \right) + C$$

$$\int \sqrt{u^2 - a^2}\, du = \frac{1}{2}\left(u\sqrt{u^2 - a^2} - a^2 \ln\left| u + \sqrt{u^2 + a^2} \right| \right) + C, u > a$$

$$\int \sqrt{u^2 + a^2}\, du = \frac{1}{2}\left(u\sqrt{u^2 + a^2} + a^2 \ln\left| u + \sqrt{u^2 + a^2} \right| \right) + C$$

Math 181

Find the arc length of the graph of $f(x) = \frac{1}{2}x^2$ from x = 0 to x = 1.

Find the surface area of the solid generated by revolving the region bounded by the graphs of $y = x^2, y = 0, x = 0, x = \sqrt{2}$ about the x-axis.