Math 181

Applications of Integration
Areas of Surfaces of Revolution and the Theorems of Pappus

If the function $f(x) \geq 0$ is continuously differentiable on [a, b], the area of the surface generated by revolving the curve $y = f(x)$ about the *x*-axis as

$S = \int_a^b 2\pi r(x)\sqrt{1+[f'(x)]^2}\,dx$, where *r(x)* is the distance from the graph to the axis of revolution.

Example: Find the area of the surface formed by revolving the graph of $y = x^2, y = 0, x = \sqrt{2}$ on the interval $[0, \sqrt{2}]$ about the y-axis.

Pappus's Theorem for Volumes

If a plane region is revolved once about a line in the plane that does not cut through the region's interior, then the volume of the solid it generates is equal to the region's area times the distance travel by the region's centroid during the revolution. If ρ is the distance from the axis of revolution to the centroid, then

$$V = 2\pi\rho A$$

Pappus's Theorem for Surface Area

If an arc of a smooth plane curve is revolved once about a line in the plane that does not cut through the arc's interior, then the area of the surface generated by the arc equals the length of the arc times the distance traveled by the arc's centroid during the revolution. If ρ is the distance from the axis of revolution to the centroid, then

$$S = 2\pi\rho L$$

Example: Set up an integral for the area of the surface generated by revolving the given curve about the indicated axis. Then find the area of the surface.

$$xy = 1 \quad 1 \le y \le 2 \quad y-axis$$