Math 181

Integrate $\int \frac{1}{x^2 - 5x + 6}\, dx$.

Solution:

$$\frac{1}{x^2 - 5x + 6} = \frac{A}{x-2} + \frac{B}{x-3}$$

$$1 = A(x-3) + B(x-2)$$

$$1 = x(A+B) + (-3A - 2B)$$

$$A + B = 0$$

$$-3A - 2B = 1$$

$$A = -1$$

$$B = 1$$

Therefore,

$$\int \frac{1}{x^2 - 5x + 6}\, dx = -\int \frac{1}{x-2}\, dx + \int \frac{1}{x-3}\, dx$$

$$= -\ln|x-2| + \ln|x-3| + C$$

Integrate $\int \frac{5x^2+20x+6}{x^3+2x^2+x}\,dx$

Solution:

$$\frac{5x^2+20x+6}{x^3+2x^2+x} = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{(x+1)^2}$$
$$5x^2+20x+6 = A(x+1)^2 + Bx(x+1) + Cx$$

To solve for A, let x = 0. This eliminates the B and C terms and yields

6 = A(1)
6 = A

To solve for C, let x = -1. This eliminates the A and B terms and yields

$$5-20+6=-C$$
$$9=C$$

The most convenient choices for x have been used, so to find B, you can use *any other value of* x along with the calculated values of A and C.

Using x = 1, A = 6, C = 9 yields

$$5+20+6 = A(4)+B(2)+C$$
$$31 = 24+2B+9$$
$$B=-1$$

Therefore,

$$\int \frac{5x^2+20x+6}{x^3+2x^2+x}dx = \int \frac{6}{x}\,dx - \int \frac{1}{x+1}\,dx + \int \frac{9}{(x+1)^2}\,dx$$
$$= 6\ln|x| - \ln|x+1| - \frac{9}{x+1} + C$$

Math 181

Find the volume of the solid generated by revolving the region bounded by
$y = \frac{2x}{x^2+1}, y = 0, x = 0, x = 3$ about the x-axis.

Math 181

Integrate $\int \frac{x-1}{(x+1)^3}\,dx$.

Solution:

$$\begin{aligned}\frac{x-1}{(x+1)^3} &= \frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{(x+1)^3} \\ x-1 &= A(x+1)^2 + B(x+1) + C\end{aligned}$$

Substituting x = -1 shows C = -2. We then differentiate both sides with respect to x, obtaining

$$1 = 2A(x+1) + B$$

Substituting x = -1 shows B = 1. We then differentiate both sides with respect to x, obtaining

$$\begin{aligned}0 &= 2A \\ A &= 0\end{aligned}$$

Therefore,

$$\int \frac{x-1}{(x+1)^3}\,dx = \int \frac{1}{(x+1)^2}\,dx - 2\int \frac{1}{(x+1)^3}\,dx$$

Math 181

Evaluate $\int \frac{e^t dt}{e^{2t} + 3e^t + 2}$