

Hyperbolic Functions

Hyperbolic sine of x :

$$\sinh x = \frac{e^x - e^{-x}}{2}$$

Hyperbolic cosine of x :

$$\cosh x = \frac{e^x + e^{-x}}{2}$$

$$\tanh x = \frac{\sinh x}{\cosh x}$$

$$\coth x = \frac{\cosh x}{\sinh x}$$

$$\operatorname{sech} x = \frac{1}{\cosh x}$$

$$\operatorname{csch} x = \frac{1}{\sinh x}$$

$$\cosh^2 x - \sinh^2 x = 1$$

Example: Derive the derivative of $f(x) = \sinh x$

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$$\frac{d}{dx}(\sinh x) = \cosh x$$

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$$\frac{d}{dx}(\tanh x) = \operatorname{sech}^2 x$$

$$\frac{d}{dx}(\coth x) = -\operatorname{csc} h^2 x$$

$$\frac{d}{dx}(\operatorname{sech} x) = -\operatorname{sech} x \tanh x$$

$$\frac{d}{dx}(\operatorname{csch} x) = -\operatorname{csch} x \coth x$$

Example: Find the derivative of y with respect to the appropriate variable.

a) $y = \ln(\cosh z)$

b) $y = \csc hx(1 - \ln \csc hx)$

Evaluate the integrals.

a) $\int 4 \cosh(3x - \ln x) dx$

b) $\int \frac{8 \sinh \sqrt{x}}{\sqrt{x}} dx$