

Applications of Integration
Fluid Pressures and Forces

Swimmers know that the deeper an object is submerged in a fluid, the greater the pressure on the object. **Pressure** is defined as the force per unit of area over the surface of the body. The pressure P of an object at depth h is $P = \omega h$ where ω is the weight-density of the liquid per unit of volume.

When calculating fluid pressure, you can use an important (and rather surprising) physical law called **Pascal's Principle**. Pascal's Principle states that the pressure exerted by a fluid at a depth h is transmitted equally *in all directions*.

The force F exerted by a fluid of constant weight density ω (per unit of volume) against a submerged vertical plane region from $y = c$ to $y = d$ is

$$F = \omega \int_c^d h(y)L(y)dy$$

where $h(y)$ is the depth of the fluid at y and $L(y)$ is the horizontal length of the region at y .

Example: A vertical gate in a dam has the shape of an isosceles trapezoid 8 feet across the top and 6 feet across the bottom, with a height of 5 feet. What is the fluid force on the gate if the top of the gate is 4 feet below the surface of the water?

Example: A circular observation window on a marine science ship has a radius of 1 foot, and the center of the window is 8 feet below water level. What is the fluid force on the window?