Math 285 Name ____________________
Fall 2009

Homework #5
Due Monday, October 28
NO late papers accepted! No excuses!

1. Decide whether the given mapping is a linear transformation. Justify your answer. If the mapping is a linear transformation, decide whether it is one-to-one, onto, both, or neither, and find a basis and dimension for Ker(T) and Rng(T).

$$T : R^3 \to P_2$$
$$T(a,b,c) = ax^2 + (2b - c)x + (a - 2b + c$$

2. Let $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ *and* $B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$ be vectors in $M_2(R)$. Show that the mapping

$$\langle A, B \rangle = a_{11}b_{11} + a_{12}b_{12} + a_{21}b_{21} + a_{22}b_{22}$$

defines an inner product in $M_2(R)$.

3. Find the change-of-basis matrix $P_{C \leftarrow B}$ from the given ordered basis B to the given ordered basis C of the vector space V.

V = P_3;

$B = \{-2 + 3x + 4x^2 - x^3, 3x + 5x^2 + 2x^3, -5x^2 - 5x^3, 4 + 4x + 4x^2\}$

$C = \{1 - x^3, 1 + x, x + x^2, x^2 + x^3\}$