Math 285 Name ________________________
Spring, 2009

Final Exam
Part 1
Show some work!

1. Use Laplace transforms to solve the differential equation below.

$$y'' + 2y' - 3y = 26e^{2t}\cos t$$
$$y(0) = 1$$
$$y'(0) = 0$$

2. Find the solution to the given IVP using the annihilator method.

$$y'' - y' - 2y = 10\sin x$$
$$y(0) = 0$$
$$y'(0) = 1$$

3. Show that $\langle (y,x),(z,w)\rangle = 2yz + 3xw$ is an inner product on R^2.

4. Is $H = \left\{ \begin{bmatrix} a \\ b \\ c \end{bmatrix} \middle| \int_0^1 \left(ax^2 + bx + c\right) dx = 0 \right\}$ a vector space? Why or why not?

5. Use the variation of parameters method to find the solution of

$$x''+4x = \tan t, x(0) = 1, x'(0) = 2$$

Set up the system of equations. Do not use the formulas. Actually integrate this system.

6. Find the general solution to the given DE.

$$\frac{dy}{dx} - \frac{1}{2x\ln x}y = 2xy^3$$

(The DE is a Bernoulli DE.)

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Final Exam
Part 2
Show some work!

1. Solve the following DE using a power series.

$$y'' + xy' + 3y = 0$$

2. Find a basis and the dimension for the row space, column space, and null space of the given matrix A.

$$A = \begin{bmatrix} 1 & 2 & -1 & 3 \\ 3 & 6 & -3 & 5 \\ 1 & 2 & -1 & -1 \\ 5 & 10 & -5 & 7 \end{bmatrix}$$

3. If $T : R^3 \rightarrow R^3$ has matrix $A = \begin{bmatrix} 2 & 3 & 1 \\ -1 & 2 & 3 \\ 4 & 1 & 6 \end{bmatrix}$ show that T^{-1} exists and find it.

4. Find the general solution to $[\sin(xy) + xy\cos(xy) + 2x]dx + [x^2 \cos(xy) + 2y]dy = 0$. (This DE is exact.)

5. Consider the linear transformation $T: R^3 \to R$ defined by

$$T(\vec{v}) = \langle \vec{u}, \vec{v} \rangle$$

Where **u** is a fixed nonzero vector in R^3.

a) Find Ker (T) and dim[Ker(T)].
b) Find Rng(T) and dim[Rng(T)].