

Exam 3  
Show some work!

1. Solve  $y'' - 3y' + 2y = e^{-4t}$ ,  $y(0) = 1$ ,  $y'(0) = 5$  using Laplace transforms.

2. Solve  $y'' + y = \sec x$  using variation of parameters.

3. Solve  $y''' + y'' = e^x \cos x$ .

4. Solve the given non-homogeneous system.

$$\begin{cases} x_1' = x_1 + 2x_2 + 5e^{4t} \\ x_2' = 2x_1 + x_2 \end{cases}$$

5. Determine whether the given matrix is diagonalizable. Where possible, find a matrix  $S$  such that  $S^{-1}AS = \text{diag}(\lambda_1, \lambda_2, \dots, \lambda_n)$ .

$$\begin{bmatrix} 0 & -2 & -2 \\ -2 & 0 & -2 \\ -2 & -2 & 0 \end{bmatrix}$$

6. Determine the general solution to the system  $\vec{x}' = A\vec{x}$  for the given matrix A.

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 5 & -7 \\ 0 & 2 & -4 \end{bmatrix}$$

7. Determine the inverse Laplace transform of

$$F(s) = \frac{2s + 3}{(s - 1)(s^2 + 1)}$$

8. Let  $T : R^4 \mapsto R^3$  be a linear transformation represented by

matrix  $A = \begin{bmatrix} 1 & -1 & 0 & 2 \\ 0 & 1 & 2 & -3 \\ 1 & 0 & 2 & -1 \end{bmatrix}$ . Find a basis for the  $Ker(T)$  and  $Rng(T)$ .