Math 285 Name ____________________
Spring, 2008

Exam 3
Show some work!

1. Determine the multiplicity of each eigenvalue and a basis for each eigenspace of the matrix. Hence determine the dimension of each eigenspace and state whether the matrix is defective or nondefective.

$$A = \begin{bmatrix} 4 & -2 & 1 \\ 2 & 0 & 1 \\ 2 & -2 & 3 \end{bmatrix}$$

(Hint: One of the eigenvalues is 2.)

2. Decide whether the given mapping is a linear transformation. Justify your answer. If the mapping is a linear transformation, decide whether it is one-to-one, onto, both, or neither, and find a basis and dimension for Ker(T) and Rng(T).

$$T : R^3 \to P_2$$
$$T(a,b,c) = ax^2 + (2b - c)x + (a - 2b + c$$

2. Continued (More space for work)

3. Find the general solution to the given differential equation.

$$y''' + 3y'' - 4y = 0$$

4. Solve the given initial-value problem.

$$x_1' = 2x_1 + x_2$$
$$x_2' = -x_1 + 4x_2$$
$$x_1(0) = 1$$
$$x_2(0) = 3$$

5. Determine whether the given matrix is diagonalizable. Where possible, find a matrix S such that $S^{-1}AS = diag(\lambda_1, \lambda_2, \cdots, \lambda_n)$.

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 7 \\ 1 & 1 & -3 \end{bmatrix}$$

6. Use the variation of parameters method to determine the general solution to the differential equation.

$$y'' - 2y' + y = 4e^{x} x^{-3} \ln x, \quad x > 0$$

7. Show that the given function are solutions of the system $\vec{x}'(t) = A(x)\vec{x}(t)$ for the given matrix A, and hence, find the general solution to the system (remember to check linear independence).

$$\vec{x}_1(t) = \begin{bmatrix} -3 \\ 9 \\ 5 \end{bmatrix}, \vec{x}_2(t) = \begin{bmatrix} e^{2t} \\ 3e^{2t} \\ e^{2t} \end{bmatrix}, \vec{x}_3(t) = \begin{bmatrix} e^{4t} \\ e^{4t} \\ e^{4t} \end{bmatrix}, A = \begin{bmatrix} 2 & -1 & 3 \\ 3 & 1 & 0 \\ 2 & -1 & 3 \end{bmatrix}$$