Math 285 Name ____________________
Spring, 2009

Exam 2
Must show some work!

1. Determine whether or not the set S is a basis for the specified vector space V.

$$V = P_4$$
$$S = \{t^4, t+3, t^3+4, t-1, t^2-5t+1\}$$

2. Decide whether or not the given set constitutes a vector space. Assume "standard" definitions of the operations.

The set Z of integers

3. Prove that if $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ is linearly independent and $\vec{v}_4$ is not in the span $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$, then $\{\vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4\}$ is linearly independent.

4. Let $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ *and* $B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$ be vectors in $M_2(R)$. Show that the mapping

$$\langle A, B \rangle = a_{11}b_{11} + a_{12}b_{12} + a_{21}b_{21} + a_{22}b_{22}$$

defines an inner product in $M_2(R)$.

5. Let $A = \begin{bmatrix} 3 & 5 & 5 & 2 & 0 \\ 1 & 0 & 2 & 2 & 1 \\ 1 & 1 & 1 & -2 & -2 \\ -2 & 0 & -4 & -2 & -2 \end{bmatrix}$. Find a basis and dimension for the rowspace, the colspace, and the nullspace of A.

B is row equivalent to A.

$$B = \begin{bmatrix} 1 & 5/3 & 5/3 & 2/3 & 0 \\ 0 & 1 & -1/5 & -1/5 & -3/5 \\ 0 & 0 & 1 & 7/2 & 3 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

Decide whether the following subset is a subspace.

a) Let S be the set of points inside and on the unit circle in R^2.

$$S = \{(x, y) \in R^2 \mid x^2 + y^2 \leq 1\}$$

b) Let S be a subset of an inner product space B, show that $S^{\perp} = \{\vec{v} \in V \mid \langle v, s \rangle = 0 \ \textit{for all} \ s \in S\}$ is a subspace of V.

6. Determine a linearly independent set of vectors that spans the same subspace of V as that spanned by the original vectors (i.e. find the minimal spanning set).

$$V = P_2$$
$$\{2 + x^2, 4 - 2x + 3x^2, 1 + x\}$$

7. Use Cramer's Rule to solve the following system.

$$\begin{cases} 2x - y + z = 2 \\ 4x + 5y + 3z = 0 \\ 4x - 3y + 3z = 2 \end{cases}$$

8. Find the change-of-basis matrix $P_{C \leftarrow B}$ from the given ordered basis B to the given ordered basis C of the vector space V.

$$V = P_2;$$
$$B = \{-4 + x - 6x^2, 6 + 2x^2, -6 - 2x + 4x^2\}$$
$$C = \{1 - x + 3x^2, 2, 3 + x^2\}$$

9. Determine whether the set of vectors $\left\{ v_1 = \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix}, v_2 = \begin{bmatrix} 1 \\ 2 \\ -4 \end{bmatrix}, v_3 = \begin{bmatrix} 7 \\ 0 \\ -10 \end{bmatrix} \right\}$ is linearly independent or linearly dependent. Find a basis of $span(v_1, v_2, v_3)$.