Math 285 Name ____________________
Spring, 2008

Exam 2
Show some work!

1. Assume that matrix A is row equivalent to matrix B. Find bases for Col Space A, Row Space A, and the Null Space A.

$$A = \begin{bmatrix} 2 & -3 & 6 & 2 & 5 \\ -2 & 3 & -3 & -3 & -4 \\ 4 & -6 & 9 & 5 & 9 \\ -2 & 3 & 3 & -4 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 2 & -3 & 6 & 2 & 5 \\ 0 & 0 & 3 & -1 & 1 \\ 0 & 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

2. Find the dimension of the subspace

$$H = \left\{ \begin{bmatrix} a - 3b + c \\ 5a + 4d \\ b - 2c - d \\ 5d \end{bmatrix} :: a, b, c, d \in R \right\}$$

3. Let $A=\begin{bmatrix} 1 & -2x & 2x^2 \\ 2x & 1-2x^2 & -2x \\ 2x^2 & 2x & 1 \end{bmatrix}$.

a) Show that $\det(A)=\left(1+2x^2\right)^3$.

b) Use the adjoint method to find A^{-1}.

4. Let V be the vector space of all real-valued functions defined on an interval [a, b], and let S denote the set of all functions in V that satisfy $f(a) = 0$. Verify that S is a subspace of V.

5. Do the rows of the matrix below span R^4?

$$\begin{bmatrix} -2 & -2 & 1 & 3 \\ 3 & 3 & 0 & -1 \\ -1 & -1 & -2 & -5 \\ 2 & 2 & 1 & 1 \end{bmatrix}$$

6. Let A and B be 3 x 3 matrices with det (A) = 3 and det (B) = - 4. Determine
   a) det (2A)

   b) det (A^{-1})

   c) det ($A^T B$)

   d) det $\left(B^{-1}AB\right)$

7. Find the change-of-basis matrix $P_{C\leftarrow B}$ from the given ordered basis B to the given ordered basis C of the vector space V.

V = P_3;

$B = \{-2 + 3x + 4x^2 - x^3, 3x + 5x^2 + 2x^3, -5x^2 - 5x^3, 4 + 4x + 4x^2\}$

$C = \{1 - x^3, 1 + x, x + x^2, x^2 + x^3\}$

8. Determine whether the set of vectors $\left\{ v_1 = \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix}, v_2 = \begin{bmatrix} 1 \\ 2 \\ -4 \end{bmatrix}, v_3 = \begin{bmatrix} 7 \\ 0 \\ -10 \end{bmatrix} \right\}$ is linearly independent or linearly dependent. Find a basis of $span(v_1, v_2, v_3)$.

9. Let S denote the subspace of $M_2(R)$ consisting of all symmetric 2 x 2 matrices. Determine a basis for S, and find dim[S]. Extend this basis for S to obtain a basis for $M_2(R)$.

10. Let $S = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \middle| a - d = 0;\ b + c = 0 \right\}$. Is S a subspace of $M_2(R)$?

11. Find the dimension of the null space of the given matrix A.

$$A = \begin{bmatrix} 1 & -1 & 2 & 3 \\ 2 & -1 & 3 & 4 \\ 1 & 0 & 1 & 1 \\ 3 & -1 & 4 & 5 \end{bmatrix}$$