Math 285 Name ____________________________

Spring, 2009

Exam 1

No Work = No Credit!

1. A tank initially contains 600 L of solution in which there is dissolved 1500 grams of chemical. A solution containing 5 g/L of the chemical flows into the tank at a rate of 6L/min and the well-stirred mixture flows out at a rate of 3L/min. Determine the concentration in the tank after one hour.

2. A simple nonlinear law of cooling states that the rate of change of temperature of an object is proportional to the square of the temperature difference between the object and its surrounding medium (you may assume that the temperature of the surrounding medium is constant). Set up and solve the initial value problem that governs this cooling process if the initial temperature is $T_0$. What happens to the temperature of the object as $T \to \infty$?

3. Use Gauss-Jordan elimination to solve the system.

$$\begin{cases} x - y - 2z = 1 \\ 2x + 3y + z = 2 \\ 5x + 4y + 2z = 4 \end{cases}$$

4. Solve each differential equation.

a) $\left[1+\ln\left(xy\right)\right]dx + xy^{-1}dy = 0$

b) $y' + 4xy = 4x^3 y^{\frac{1}{2}}$

c) $\frac{d^2y}{dx^2} = \frac{1}{x}\left(\frac{dy}{dx} + x^2 \cos x\right), x > 0$

d) $2x(y+2x)y' = y(4x-y)$

e) $y^{-\frac{2}{3}}\frac{dy}{dx}+\frac{3}{x}y^{\frac{1}{3}}=\frac{12}{\sqrt{1+x^2}}$

5. Let $A\begin{bmatrix} -2 & 4 & 2 & 6 \\ -1 & -1 & 5 & 0 \end{bmatrix}$, $B = \begin{bmatrix} -3 & 0 \\ 2 & 2 \\ 1 & -3 \\ 0 & 1 \end{bmatrix}$, $C = \begin{bmatrix} -5 \\ -6 \\ 3 \\ 1 \end{bmatrix}$ and $r = -4$. Compute the given expression, if possible.

a) $rA - B^T$

b) $AB$ and tr(AB)

c) $(AC)(AC)^T$

6. Solve the initial value problem.

$$\frac{dy}{dx} - (\sin x)y = e^{-\cos x}, \quad y(0) = \frac{1}{e}$$