Math 285 Name ______________________
Spring, 2008

Exam 1
Show some work!

1. Identify the type of differential equation and then find the general solution.

a) $\frac{dy}{dx} = \frac{-2xy}{x^2 + 2y}$

b) $\left(y^2 + 3xy + x^2\right)dx - x^2 dy = 0$

c) $\frac{dy}{dx}+\frac{2e^{2x}}{1+e^{2x}}y=\frac{1}{e^{2x}-1}$

d) $\frac{dy}{dx} + \frac{1}{x} y = \frac{25x^2 \ln x}{2y}$

e) $e^{2x+y}dy - e^{x-y}dx = 0$

f) $\frac{dy}{dx} + \frac{2x}{1+x^2} y = \frac{1}{(1+x^2)^2}$

2. A simple nonlinear law of cooling states that the rate of change of temperature of an object is proportional to the square of the temperature difference between the object and its surrounding medium (you may assume that the temperature of the surrounding medium is constant). Set up and solve the initial-value problem that governs this cooling process if the initial temperature is T_0. What happens to the temperature of the object as $T \rightarrow \propto$?

3. A tank initially contains 600 L of solution in which there is dissolved 1500 grams of chemical. A solution containing 5 g/L of the chemical flows into the tank at a rate of 6 L/min, and the well-stirred mixture flows out at a rate of 3 L/minute. Determine the concentration in the tank after one hour.

4. Use elementary row operations to reduce the given matrix to row-echelon form, and hence determine the rank of the matrix.

$$\begin{bmatrix} 2 & -2 & -1 & 3 \\ 3 & -2 & 3 & 1 \\ 1 & -1 & 1 & 0 \\ 2 & -1 & 2 & 2 \end{bmatrix}$$

5. If $A = \begin{bmatrix} 2 & -1 & 0 \\ 3 & 1 & 2 \\ -1 & 1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & 1 \\ -1 & 1 & 0 \end{bmatrix}$, and $C = \begin{bmatrix} -1 & -1 & 1 \\ 1 & 2 & 3 \\ -1 & 1 & 0 \end{bmatrix}$.

a) Find the matrix D such that $2A + B - 3C + 2D = A + 4C$.
b) Find AB.
c) Find BA.
d) Find A^T