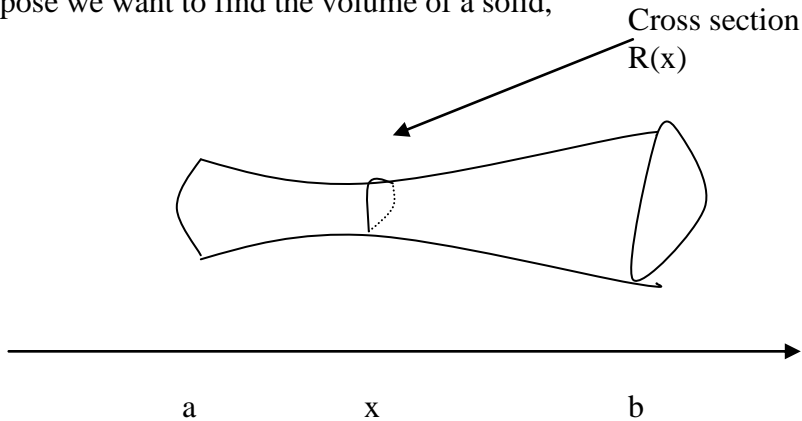


Applications of Integration
Finding Volumes

Suppose we want to find the volume of a solid,



The cross section of the solid at each point x in the interval $[a, b]$ is a region $R(x)$ of area $A(x)$.

If A is a continuous function of x , use it to define and calculate the volume of the solid by integrating $A(x)$ from a to b .

The volume of a solid of known integrable cross-section area $A(x)$ from $x = a$ to $x = b$ is the integral of A from a to b :

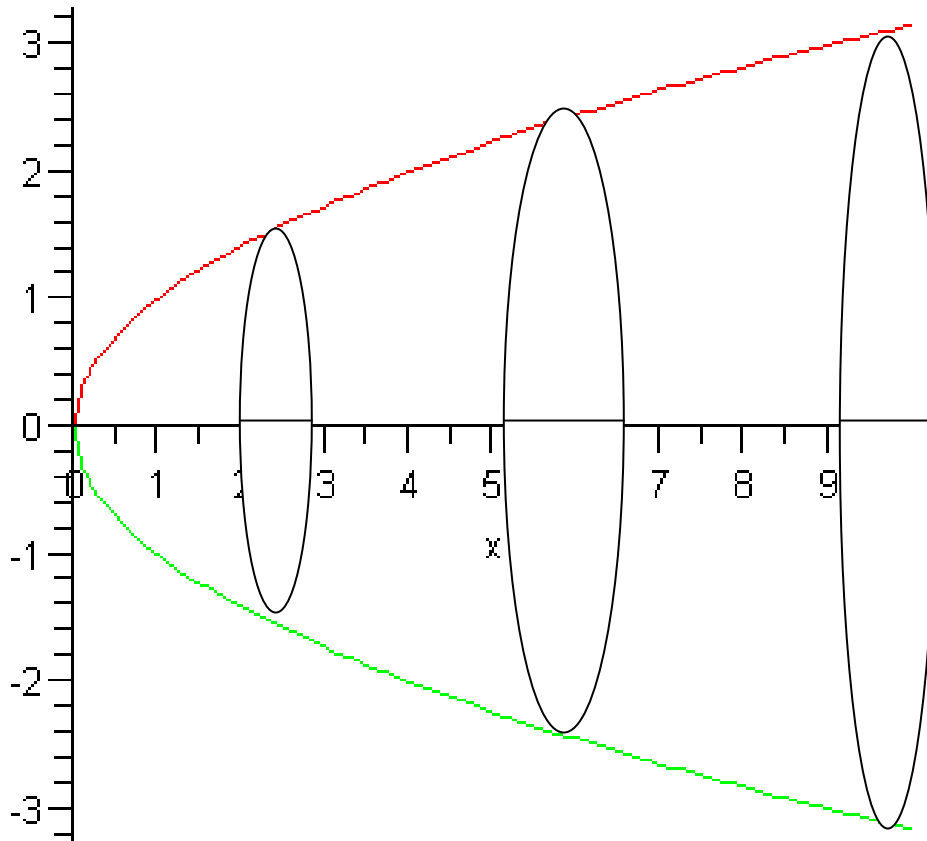
$$V = \int_a^b A(x) dx$$

HOW TO FIND THE VOLUME BY SLICING

1. Sketch the solid and a typical cross section.
2. Find a formula for $A(x)$.
3. Find the limits of integration.
4. Integrate $A(x)$ to find the volume.

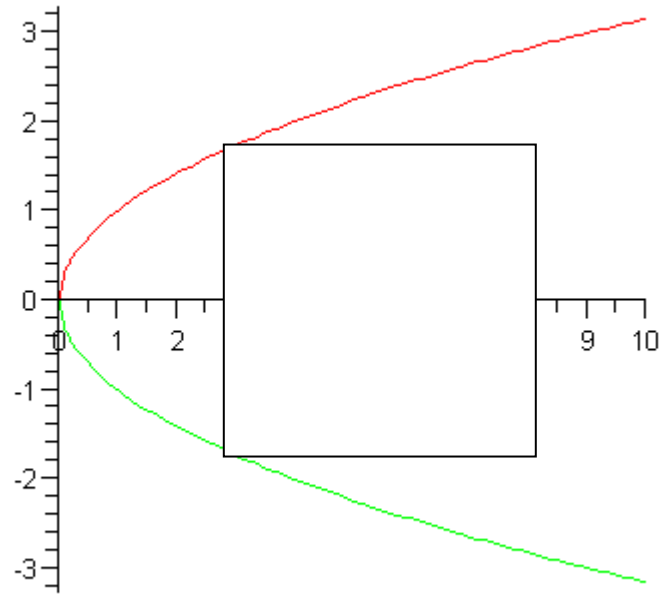
Example: The solid lies between planes perpendicular to the x-axis at $x = 0$ and $x = 4$. The cross sections perpendicular to the x-axis between these planes run from the parabola $y = -\sqrt{x}$ and $y = \sqrt{x}$.

a) the cross sections are circular disks with diameters in the xy-plane



$$\begin{aligned}
 A &= \pi r^2 \\
 &= \pi(\sqrt{x})^2 = \pi x \\
 V &= \pi \int_0^4 x dx
 \end{aligned}$$

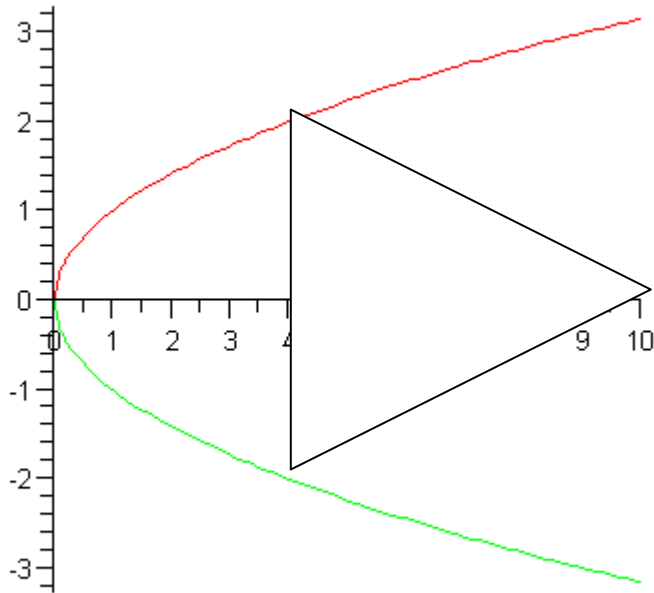
b) cross sections are squares with bases in the xy-plane



$$A = s^2 = (2\sqrt{x})^2 = 4x$$

$$V = \int_0^4 4x dx$$

b) the cross sections are equilateral triangles with bases in the xy-plane



$$A = \frac{1}{2}bh = \frac{1}{2}(2\sqrt{x})\sqrt{3x} = \sqrt{3x}$$

$$V = \int_0^4 \sqrt{3x} dx$$

Disk Method

Solids of revolution are solids whose shapes can be generated by revolving plane regions about an axis.

Cross sections are circular.

$A(x) = \pi[R(x)]^2$ where $R(x)$ is the radius of the disk.

If the solid has a hole in it,

$A = \pi[R^2(x) - r^2(x)]$ where $R(x)$ is the outer radius and $r(x)$ is the inner radius.

The approximating rectangle is perpendicular to the axis of revolution.

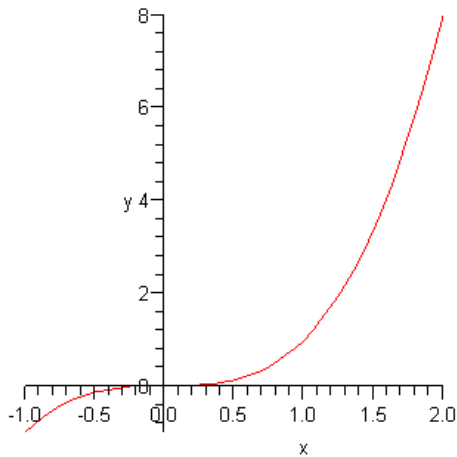
Example: Find the volume of the solid generated by revolving the region about the given axis.

$$y = x^3$$

a) $y = 0$

$$x = 2$$

about the x-axis

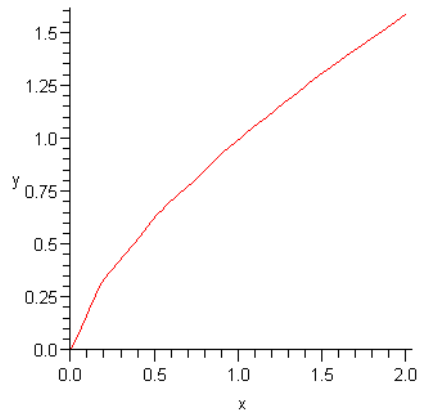


$$x = y^{\frac{3}{2}}$$

b) $x = 0$

$$y = 2$$

about the y-axis

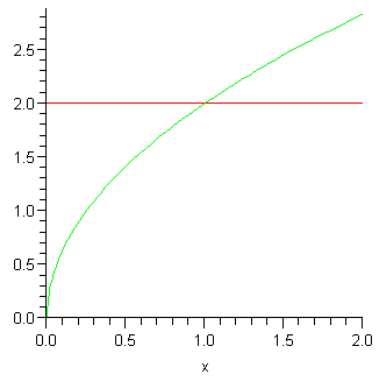


$$y = 2\sqrt{x}$$

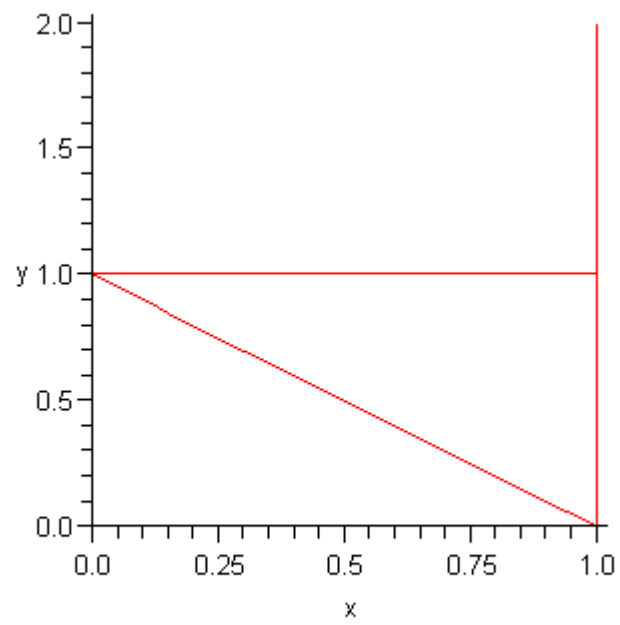
c) $y = 2$

$$x = 0$$

about the x-axis



c) the region bounded by the triangle with vertices $(0, 1)$, $(1, 0)$, and $(1, 1)$ revolved about the y -axis



$$y = 2x$$

e) $y = 0$

$$x = 1$$

about line $x = 2$

Shell Method

Consider a representative rectangle where w is the width of the rectangle, h is the height of the rectangle and p is the distance between axis of revolution and the center of the rectangle. When this rectangle is revolved about its axis of revolution, it forms a cylindrical shell of thickness x .

The volume of the shell is

$$V = 2\pi \int_a^b p(x)h(x)dx$$

where $p(x)$ is the distance from axis of revolution to the center of the rectangle and $h(x)$ is the height of the rectangle.

The approximating rectangle is parallel to the axis of revolution.

Example: Find the volume of the solid generated by revolving the region about the given axis.

$$y = x - x^3$$

a) $x = 0$

$$y = 0$$

about y - axis

$$y = 2 - x^2$$

b) $y = x^2$

$$x = 0$$

about y-axis

Comparison of Disk and Shell Method

Disk Method

Rectangle is perpendicular to
axis of revolution

Shell Method

Rectangle is parallel to
axis of revolution

THIS TELLS YOU WHAT YOU ARE INTEGRATING
WITH RESPECT TO.

Multiply by π

Multiply by 2π

Example: Find the volume of the solid formed by revolving the region bounded by the

$$y = x^3 + x + 1$$

graphs $y = 1$ about the line $x = 2$.

$$x = 1$$