

Symmetry

Symmetry with respect to the Polar Axis (x-axis)

Replace θ by $-\theta$.

If an equivalent equation results, the graph is symmetric with respect to the polar axis.

Symmetry with respect to the line $\theta = \frac{\pi}{2}$ (y-axis)

Replace θ by $\pi - \theta$.

If an equivalent equation results, the graph is symmetric with respect to the line $\theta = \frac{\pi}{2}$.

Symmetry with respect to the pole (origin)

Replace r by $-r$.

If an equivalent equation results, the graph is symmetric with respect to the pole.

Cardioids

$$r = a(1 \pm \cos \theta)$$

$$r = a(1 \pm \sin \theta)$$

$$a > 0$$

The graph of a cardioid passes through the pole.

Limaçons Without an Inner Loop

$$r = a \pm b \cos \theta$$

$$r = a \pm b \sin \theta$$

$$a > 0, b > 0, a > b$$

The graph of a limaçon without an inner loop does not pass through the pole.

Limaçons With An Inner Loop

$$r = a \pm b \cos \theta$$

$$r = a \pm b \sin \theta$$

$$a > 0, b > 0, a < b$$

The graph of a **limaçon** with an inner loop will pass through the pole twice.

Roses

$$r = a \cos(n\theta)$$

$$r = a \sin(n\theta)$$

$$a \neq 0$$

If $n \neq 0$ is even, the rose has $2n$ petals.

If $n \neq \pm 1$ is odd, the rose has n petals.

Lemniscates

$$r^2 = a^2 \sin(2\theta)$$

$$r^2 = a^2 \cos(2\theta)$$

$$a \neq 0$$

Lemniscates have graphs that are propeller shaped.