

Math 181
Spring, 2008

Name _____

Final Exam
No Calculator!
Be careful with time!

Evaluate each of the following integrals.

1. $\int \frac{5x^2 + 20x + 6}{x^3 + 2x^2 + x} dx$

$$2. \int \frac{x^2}{\sqrt{16-x^6}} dx$$

3. $\int e^{2x} \cos(2x) dx$

4. $\int \frac{dx}{x^2 \sqrt{9-x^2}}$

5. $\int \frac{\ln(x)}{x^2} dx$

$$6. \int \frac{dx}{(x^2 + 1)^{3/2}}$$

Math 181
Spring, 2008

Name _____

Final Exam
Part 2
No work = No credit!

1. **Set up but do not evaluate** the integral that represents the area inside the circle $r = 12 \cos \theta$ and outside the cardioid $r = 4 + 4 \cos \theta$.

2. **Set up the integral but do not evaluate the integral** to find the volume of the solid generated by revolving the region bounded by $y = x$ and $y = x^2$ about the line $x = -1$.

3. Evaluate $\int_{-\infty}^{\infty} \frac{e^x}{1+e^{2x}} dx$

4. Find a power series for $g(x) = \arctan x$ centered at 0 (zero). Remember that

$$\frac{d}{dx} \arctan x = \frac{1}{1+x^2}. \quad \mathbf{8 \text{ points}}$$

5. Find the interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{n^3}{3^n} (x+1)^n$ **6 points**

6. Find an equation of the plane containing the points $(2,1,1)$, $(0,4,1)$, and $(-2,1,4)$.

7. Find an equation of the plane through the points $(3,2,1)$ and $(3,1,-5)$ and is perpendicular to the plane $6x + 7y + 2z = 10$.

8. Find a set of parametric equations of the line that passes through the point $(2, 3, 4)$ and is perpendicular to the plane $3x + 2y - z = 6$.

9. Find a rectangular equation for the graph represented by the cylindrical equation $r^2 \cos(2\theta) + z^2 + 1 = 0$. **5 points**

10. Classify and sketch the surface given by $x - y^2 - 4z^2 = 0$. Identify the traces. **5 points**

11. Let $\vec{u} = \langle 3, -2, 1 \rangle$, $\vec{v} = \langle 2, -4, -3 \rangle$, and $\vec{w} = \langle -1, 2, 2 \rangle$. **2 points each**

- a) Find $|\vec{u}|$.
- b) Find the angle between \mathbf{u} and \mathbf{v} .
- c) Determine a unit vector perpendicular to the plane containing \mathbf{v} and \mathbf{w} .
- d) Determine the projection of \mathbf{w} onto \mathbf{u} .
- e) Show that $\vec{u} \times \vec{v} = -(\vec{v} \times \vec{u})$.
- f) Find the volume of the solid whose edges are \mathbf{u} , \mathbf{v} , and \mathbf{w} .
- g) Find the work done in moving an object along the vector \mathbf{u} , if the applied force is \mathbf{w} .

12. **Set up, but do not evaluate**, the integral that will find the area of the region inside the large loop and outside the small loop of $r = 1 - 2 \sin \theta$. **6 points**

13. Solve the differential equation using power series.

$$y'' - xy = 0$$

14. A conical tank is resting on its base, which is at 10 feet below ground level. The tank has radius of 5 feet and height of 10 feet. The tank is half full of oil weighing 75 pounds per cubic foot. Find the work done in emptying the tank. **8 points**