Math 181

Applications of Integration
Work

We say that work is done by a force when it moves an object. If the force applied to the object is constant, we have the following definition of work.

If an object is moved a distance D in the direction of an applied constant force F, then the work W done by the force is defined as $W = FD$.

A force can be thought of as a push or a pull; a force changes the state of rest or state of motion of a body.

If a variable force is applied to an object, calculus is needed to determine the work done, because the amount of force changes as the object changes position. For instance, the force required to compress a spring increases as the spring is compressed.

If an object is moved along a straight line by a continuously varying force *F(x)*, then the work W done by the force as the object is moved from $x = a$ to $x = b$ is

$$W = \int_a^b F(x)dx$$

Hooke's Law

The force F required to compress or stretch a spring (within its elastic limits) is proportional to the distance that the spring is compressed or stretched from its original length. That is,

$$F = kd$$

where the constant of proportionality *k* (the spring constant) depends on the specific nature of the spring.

Example: A force of 750 pounds compresses a spring 3 inches from its natural length of 15 inches. Find the work done in compressing the spring an additional 3 inches.

Example: A spherical tank of radius 8 feet is half full of oil that weighs 50 pounds per cubic foot. Find the work required to pump oil out through a hole in the top of the tank.

Example: A 20-foot chain weighing 5 pounds per foot is lying coiled on the ground. How much work is required to raise one end of the chain to a height of 20 feet so that it is fully extended?

Example: An open tank has the shape of a right circular cone. The tank is 8 feet across the top and 6 feet high. How much work is done in emptying the tank by pumping the water over the top edge? The water weighs 62.4 pounds per cubic foot.