Math 181
Fall, 2008

Lab #2

Most wines are produces entirely from grapes grown in a single year. Sherry, however, is a complex mixture of older wines with new wines. This is done with a sequence of barrels (called a solera) stacked on top of each other.

The oldest wine is in the bottom tier of barrels and the newest is in the top tier. Each year, half of each barrel in the bottom tier is bottled as sherry. The bottom barrels are then refilled with the wine from the barrels above. This process is repeated throughout the solera, with new wine being added to the top barrels. A mathematical model for the amount of n-year old wine that is removed from a solera (with k tiers) each year is

$$f(n,k) = \binom{n-1}{k-1}\left(\frac{1}{2}\right)^{n+1} \qquad k \le n$$

a) Consider a solera that has tiers numbered k = 1, 2, 3, 4, and 5. In 1990 (n = 1), half of each barrel in the top tier (tier 1) was refilled with new wine. How much of this wine as removed from the solera in 1991? In 1992? In 1993? … In 2005? During which year(s) was the greatest amount of the 1990 wine removed from the solera?

b) In part (a), let a_n be the amount of 1990 wine that is removed from the solera in year n. Evaluate

$$\sum_{n=0}^{\infty} a_n .$$

For further information, see the article "Finding Vintage Concentrations in a Sherry Solera" by Rhodes Peel and John T. MacQueen in the UMAP Modules.

Lab # 2 is due Wednesday, November 26.