Math 180

A moving body's average speed during an interval of time is found by dividing the distance covered by the time elapsed.

The average rate of change of $y = f(x)$ with respect to x over the interval $[x_1, x_2]$ is

$$\frac{\Delta y}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{f(x+h) - f(x)}{h}, h \neq 0$$

Geometrically, the rate of change of f over $[x_1, x_2]$ is the slope of the line through the points $P(x_1, f(x_1))$ and $Q(x_2, f(x_2))$.

Informal definition of limit:

Let $f(x)$ be defined on an open interval about x_0, except possibly at x_0 itself. If f(x) gets arbitrarily close to L (as close to L as we like) for all x sufficiently close to x_0, we say that f approaches the limit L as x approaches x_0, and we write

$$\lim_{x \to x_0} f(x) = L$$

Example: Let $f(x) = \dfrac{x^2 - 2x - 3}{x^2 - 4x + 3}$.

a) Make a table of the values of f at x = 2.9, 2.99, 2.999, 3.1, 3.01, 3.001. Use the table to estimate $\lim_{x \to 3} f(x)$.
b) Support your answer graphically.
c) Find $\lim_{x \to 3} f(x)$ algebraically.

Theorem: If L, M, c and k are real numbers and

$$\lim_{x \to c} f(x) = L \quad and \quad \lim_{x \to c} g(x) = M, \text{ then}$$

a) $\lim_{x \to c}[f(x) \pm g(x)] = \lim_{x \to c} f(x) \pm \lim_{x \to c} g(x) = L \pm M$

b) $\lim_{x \to c}[f(x) \cdot g(x)] = \lim_{x \to c} f(x) \cdot \lim_{x \to c} g(x) = L \cdot M$

c) $\lim_{x \to c}[k \cdot f(x)] = k \lim_{x \to c} f(x) = kL$

d) $\lim_{x \to c}\left[\frac{f(x)}{g(x)}\right] = \frac{\lim_{x \to c} f(x)}{\lim_{x \to c} g(x)} = \frac{L}{M}, \quad M \neq 0$

Sandwich Theorem:

Suppose that $g(x) \le f(x) \le h(x)$ for all x in some open interval containing c, except possible at $x = c$ itself. Suppose also that

$$\lim_{x \to c} g(x) = \lim_{x \to c} h(x) = L.$$

Then $\lim_{x \to c} f(x) = L$.

Example: Find each of the following limits:

$$\lim_{x \to 9} \frac{\sqrt{x} - 3}{x - 9}$$

b) $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ where $f(x) = \frac{1}{3-x}$.

c) $\lim_{x \to 4} \frac{4-x}{5-\sqrt{x^2+9}}$