

Math 180  
Spring, 2005

Name \_\_\_\_\_

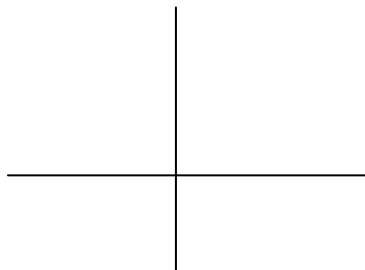
Exam 1  
Calculator OK!  
No work = No Credit!

1. Let  $y = f(x) = -x^2 + 3$ . Find the equation of the tangent line to the graph of  $y$  at the point  $(-1, 2)$ . Graph both the function and the tangent line. **8 points**

2. Let  $f$  be an even function and let  $g$  be an odd function. Prove that  $f \circ g$  is an even function. **5 points**

3. Let  $f(x) = \frac{\sqrt{x^2 - 9}\sqrt{25 - x^2}}{x - 4}$ . Determine the intervals at which  $f$  is continuous. **5 points**

4. A rectangle is bounded by the x-axis and the semi-circle  $y = \sqrt{25 - x^2}$ . Write the area  $A$  of the rectangle as a function of  $x$ . **4 points**



5. Use the  $\varepsilon/\delta$  definition to prove  $\lim_{x \rightarrow 2} (3x + 2) = 8$ . **5 points**

6. Find the constants  $a$  and  $b$  so that the given function is continuous everywhere.  
Use the definition. **8 points**

$$f(x) = \begin{cases} ax + 3, & x > 5 \\ 8, & x = 5 \\ x^2 + bx + 1, & x < 5 \end{cases}$$

7. Show that the equation  $x + \sin x = \frac{1}{\sqrt{x+3}}$  has at least one solution in the interval  $(0, \pi)$ . **4 points**

8. Given that  $1 - \frac{x^2}{4} \leq u(x) \leq 1 + \frac{x^2}{2}$  for all  $x \neq 0$ , find  $\lim_{x \rightarrow 0} u(x)$ . **4 points**

9. If functions  $f(x)$  and  $g(x)$  are continuous on the closed interval  $[0, 1]$ , could  $\frac{f(x)}{g(x)}$  possibly be discontinuous at a point of  $[0, 1]$ ? Give reasons for your answer. **4 points**

10. Find the following limits analytically. Plus or minus infinity is an acceptable answer.

a)  $\lim_{x \rightarrow 0} \frac{1 - \sqrt{1 - x^2}}{x}$

b)  $\lim_{x \rightarrow 3} \frac{x^2 - 9}{x^2 + 2x - 3}$

c)  $\lim_{x \rightarrow 2} \lfloor x \rfloor$

d)  $\lim_{x \rightarrow 0} \left( \frac{1}{x} - \frac{1}{x^2 + 1} \right)$

e)  $\lim_{h \rightarrow 0} \frac{(3+h)^2 - 3}{h}$

11. Let  $f(x) = \begin{cases} \sqrt{-x} & \text{if } x < 0 \\ 3-x & \text{if } 0 \leq x < 3 \\ (x-3)^2 & \text{if } x \geq 3 \end{cases}$ .

a) Evaluate each limit, if it exists.

$$\lim_{x \rightarrow 3^+} f(x)$$

$$\lim_{x \rightarrow 3^-} f(x)$$

$$\lim_{x \rightarrow 3} f(x)$$

$$\lim_{x \rightarrow 0^+} f(x)$$

$$\lim_{x \rightarrow 0^-} f(x)$$

$$\lim_{x \rightarrow 0} f(x)$$

- b) Where is  $f$  discontinuous?  
 c) Sketch the graph of  $f$ .