

An Introduction to Problem Solving

Most people do not have an accurate picture of mathematics. For many of you, mathematics is a torture of tests, homework, and problems, problems, problems. The very word “*problems*” suggests unpleasantness and anxiety. But mathematics is not “problems.”

Some people view mathematics as a set of formulas to be applied to a list of problems at the end of textbook chapters. Formulas in algebra, trigonometry, and calculus are incredible useful. But, in this course, you will see that mathematics is a network of intriguing ideas—not a dry, formal list of techniques.

I want you to discover what mathematics really is and to become a fan. However, if you are not intrigued with the romance of the subject, that’s fine too, because at least you will have a firmer understanding of what it is you are judging. **Mathematics is a living, breathing, changing organism with many facets to its personality. It is creative, powerful, and even artistic.**

Part of the power of mathematics lies in its inexorable quest for elegance, symmetry, order, and grace. Seeking pattern, order, and understanding is a transforming process that mathematics can help us develop.

Fun and Games: An Introduction to Rigorous Thought

Fun and games – rigorous thought: only in mathematics. Who says that deep ideas and important consequences come only from hard work? When I really get down to it, when I think of mathematics, I think about rigorous thought along with fun and games, and I hope that one day you will too.

Start with two important and fundamental observations:

- 1) Logical and creative thinking are involved in mathematics.
- 2) Thinking can be fun.

By grappling with conundrums serious or otherwise, we can discover significant concepts. As we grope for solutions to silly stories, we begin to develop effective strategies for serious thinking.

You now have a mission. Your mission, should you decide to accept it, is to read the following stories and attempt to answer the questions they raise. The only rules are as follows:

- 1) Make an earnest attempt to resolve each story.
- 2) Think about each story, and be creative.
- 3) Don't give up: When stuck, look at the story in a different way.
- 4) If you become frustrated, stop working, move on, and then return to the story later.
- 5) Share these stories with your family, friends, or random people.
- 6) **HAVE FUN!**

That's a Meanie Genie

On an archeological dig near the highlands of Tibet, Alley discovered an ancient oil lamp. Just for laughs she rubbed the lamp. She quickly stopped laughing when a huge puff of magenta smoke spouted from the lamp, and an ornery genie named Murray appeared. Murray, looking at the stunned Alley exclaimed, "Well, what are you staring at? Okay, okay, you've found me; you get your three wishes. So what will they be?" Alley, although in shock, realized what an incredible opportunity she had. Thinking quickly, she said, "I'd like to find the Rama Nujan, the jewel that was first discovered by Hardy the High Lama." "You got it," replied Murray, and instantly nine identical-looking stones appeared. Alley looked at the stones and was unable to differentiate any one from the others.

Finally, she said to Murray, "So, where is the Rama Nujan?" Murray explained, "It is embedded in one of these stones. You said you wished to find it. So now you have to find it. Oh, by the way, you may take only one of the stones with you, so you had best be careful how you choose!" "But they look identical to me. How will I know which one has the Rama Nujan in it?" Alley questioned. "Well, eight of the stones weigh the same, but the stone containing the jewel weighs slightly more than the others," Murray responded with a devilish grin.

Alley, now getting annoyed, whispered under her breath, "Gee, I wish I had a balance scale." Suddenly, a balance scale appeared. "That was wish two!" declared Murray. "Hey, that's not fair!" Alley cried. "You want to talk fair? You think it's fair to be locked in a lamp for 1,729 years? You know you can't get cable in there, and there's no room for a satellite dish! So don't talk to me about fair," Murray proclaimed, "Hey, I want to help you out, so let me give you a tip: That balance scale may be used only once." "What? Only once? she said, thinking out loud. "I wish I had another balance scale." ZAP! Another scale appeared. "Okay, kiddo, that was wish number three." Murray snickered. "Hey, just one minute," Alley said, now regretting not having asked for one million dollars or something more standard. "Well at least this new scale works correctly, right?" "Sure, just like the other one. You may use it only once." "Why?" Alley inquired. "Because it is a 'wished' scale. That means that you can use it only once since it was only one wish. It's just like you cannot wish for a hundred wishes." "You are a very obnoxious genie." "Hey, don't blame me. I don't make up the rules, lady. I just follow them."

So Alley may use each of the two balance scales exactly once. How would you find the Rama Nujan?

Mission “Possible”

It really is an interesting story. As related by Carl Friedrich Gauss (1777-1855) many years after the fact, he was confronted one day in elementary school with a busy work assignment. The problem was to add up the first one hundred counting numbers. Although easy enough, the problem was tedious and, like too many arithmetic assignments of both that and this day, it seems to have been assigned only to keep the pupils quiet while providing a respite for the teacher, one Herr Buttner.

A quick solution to the problem was quite well known to mathematicians of the time, but certainly not to most 10-year-old students. While his peers laboriously undertook to find the sum, Gauss reportedly wrote the answer within seconds and handed his slate, face down, to the teacher. Nothing was left for the young Gauss to do but sit quietly weather the unrelenting glare of his teacher until all of the other students had finished their work. One can imagine Herr Buttner’s astonishment when, after scrutinizing all of the answers turned in by the end of the class, only Gauss’ solution was correct.

Your mission is to find the sum of the first 100 counting numbers.

$$1 + 2 + 3 + 4 + 5 + \dots + 96 + 97 + 98 + 99 + 100$$

Lessons for Life

1. Just do it.
2. Make mistakes and fail, but never give up.
3. Keep an open mind.
4. Explore the consequence of new ideas.
5. Seek the essential.
6. Understand the issue.
7. Understand simple things deeply.
8. Break a difficult problem over easier ones.
9. Examine issues from several points of view.
10. Look for patterns and similarities.

The First Day of Class

It is the first day of class in your new math class. Your instructor wants you and all your fellow students to introduce yourself to each other. So the class session begins with each student shaking hands with all the other students. Assuming that all 30 students (including yourself) are present, how many handshakes must take place before the class begins?

Sherlock Holmes

Sherlock Holmes has been called in to investigate a sensational robbery case. Inspector Lestrade of Scotland Yard has rounded up three suspects, but is stumped by their devious backgrounds. Reluctantly, he has called on Holmes to solve his dilemma. Holmes recognizes the three notorious suspects immediately: Mikey, although an inveterate rogue, is unscrupulously honest; he cannot lie. Raymond, ever the charming rascal, is incapable of telling the truth. Jackie...well, Jackie is equally adept at telling the truth and hiding the truth, depending on the situation. "Have no fear, Lestrade, it should be quite simple to determine the guilty party. I need only ask them each one simple question." Turning to the thugs, Holmes asked, "Very well, we know one of you is guilty, so which of you is responsible for this despicable crime?" Their responses were quick and earnest:

Mikey: "I don't know who did it!"

Jackie: "I don't know who did it!"

Raymond: "I know who did it!"

"Hmmm..." said Holmes, "So _____ did it. Gentlemen, take him away." Solve the mystery.

Can you solve the following puzzler?

A man decides to visit a series of casinos. It costs \$1.00 to enter the first one. He plays various games of chance and ends up losing half of his money. Upon leaving, he pays a \$1.00 tip to the croupier. Visiting another casino, he again pays a \$1.00 entrance charge and again loses half of his money. Still generous, he again tips the croupier \$1.00. And so he makes his way to the third establishment with a similar \$1.00 price of admission. Again, his unlucky streak continues and he loses half of his money. "Oh well," he says to third sympathetic croupier, "it's just not my lucky day. You might as well have a tip too, though it's my last dollar." The question is how much money did he start the day with?

Worm Journey

A worm is at the bottom of a 12-foot wall. Every day it crawls up 3 feet, but at night it slips down 2 feet. How many days does it take the worm to get to the top of the wall?

Life on the Farm

Emil and Olive lived on a farm with their father, Gordon. One day, Emil asked his father, “Dad, what happened to that cat we used to have?” Olive overheard this, and said, “Yeah, and we used to have a horse too.”

Gordon replied, “That tomcat and that old nag were no use. I traded them for our new goat.”

Emil said, “Hey, that sounds like a good cryptarithmic problem. Come on, Olive, let’s go see if we can solve it.”

Each letter stands for a different digit, 0 through 9. No two letters stand for the same digit. Determine which digit each letter represents.

$$\begin{array}{r} \text{T O M} \\ + \text{N A G} \\ \hline \text{G O A T} \end{array}$$